Unit – I

OPTICS - Diffraction

Introduction

The wave nature of light is first confirmed by the phenomenon of interference. Further it is confirmed by the phenomenon of diffraction. The word ‘diffraction’ is derived from the Latin word diffractus which means break to piece. When the light waves encounter an obstacle, they bend round the edges of the obstacle. The bending is predominant when the size of the obstacle is comparable with the wavelength of light. The bending of light waves around the edges of an obstacle is diffraction. It was first observed by Gremaldy.

1. Diffraction

Definition:-

When the light falls on the obstacle whose size is comparable with the wavelength of light then the light bends around the obstacle and enters in the geometrical shadow. This bending phenomenon of light is called diffraction.

When the light is incident on an obstacle AB, their corresponding shadow is completely dark on the screen. Suppose the width of the slit is comparable to the wavelength of light, then the shadow consists of bright and dark fringes. These fringes are formed due to the superposition of bended waves around the corners of an obstacle. The amount of bending always depends on the size of the obstacle and wavelength of light used.

2. Types of diffraction

The diffraction phenomena are classified into two ways

I. Fresnel diffraction
II. Fraunhofer diffraction

I. Fresnel diffraction:-

In this diffraction the source and screen are separated at finite distance. To study this diffraction lenses are not used because the source and screen separated at finite distance. This diffraction can be studied in the direction of propagation of light. In this diffraction the incidence wave front must be spherical of cylindrical.

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II. Fraunhofer diffraction:-

In this diffraction the source and screen are separated at infinite distance. To study this diffraction lenses are used because the source and screen separated at infinite distance. This diffraction can be studied in any direction. In this diffraction the incidence wave front must be plane.

3. Fraunhofer single slit diffraction:-

Let us consider a slit AB of width ‘e’. Let a plane wave front WW’ of monochromatic light of wavelength λ is incident on the slit AB. According to Huygens principle, every point on the wavefront is a source of secondary wavelets. The wavelets spread out to the right in all directions. Those secondary wavelets travelling normal to the slit are brought to focus at point P₀ on the screen by using the lens. Those secondary wavelets which are focused at P₀ have no path difference. Hence at point P₀ the intensity is maximum and is known as central (or principal) maximum. The secondary wavelets traveling at an angle θ with the normal are focused at point P₁.

Intensity at point P₁ depends upon the path difference between the wavelets A and B reaching to point P₁. To find the path difference, a perpendicular AC is drawn to B from A.

The path difference between the wavelets from A and B in the direction of θ is

\[ \text{path difference} = BC = AB \sin \theta = e \sin \theta \]

\[ \text{phase difference} = \frac{2\pi}{\lambda} \text{(path difference)} = \frac{2\pi(e \sin \theta)}{\lambda} \]

Let the width of the slit is divided into ‘n’ equal parts and the amplitude of the wave front each part is ‘a’. Then the phase difference between any two successive waves from these parts would be

\[ \frac{1}{n} \text{(phase difference)} = \frac{1}{n} \left( \frac{2\pi e \sin \theta}{\lambda} \right) = d \]
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OPTICS - Diffraction

Engineering Physics

Using the vector addition method, the resultant amplitude \( R \) is

\[
R = \frac{\alpha \sin \frac{nd}{2}}{\sin \frac{d}{2}} = \frac{n \sin \frac{1}{2} \left( \frac{2\pi \sin \theta}{\lambda} \right)}{\sin \frac{1}{2} \left( \frac{2\pi \sin \theta}{\lambda} \right)} = \frac{\alpha \sin \frac{\pi e \sin \theta}{\lambda}}{\sin \frac{\pi e \sin \theta}{n\lambda}}
\]

\[
R = \frac{\alpha \sin \alpha}{\sin \frac{\alpha}{n}} \quad \text{where} \quad \alpha = \frac{\pi e \sin \theta}{\lambda}
\]

\[
R = \frac{n \alpha \sin \alpha}{\alpha} \quad \text{when} \quad \frac{\alpha}{n} \text{ is very less} \quad \sin \left( \frac{\alpha}{n} \right) = \frac{\alpha}{n}
\]

\[
R = A \frac{\sin \alpha}{\alpha} \quad \therefore \ n\alpha = A
\]

Therefore resultant intensity \( I = R^2 = A^2 \left( \frac{\sin \alpha}{\alpha} \right)^2 \)

Principal maximum:

The resultant amplitude \( R \) can be written as

\[
R = \frac{A}{\alpha} \left( \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots \right)
\]

\[
= \frac{A}{\alpha} \left( 1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots \right)
\]

\[
= A \left( 1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots \right)
\]

In the above expression if the negative values of \( \alpha = 0 \), the resultant amplitude is maximum

\[
R = A
\]

Then

\[
I_{\text{max}} = R^2 = A^2
\]

\[
\alpha = \frac{\pi e \sin \theta}{\lambda} = 0
\]

\[
\sin \theta = 0
\]

\[
\theta = 0
\]

For \( \theta = 0 \) value the resultant intensity is maximum at \( P_0 \) and is known as principal maximum.

Minimum intensity positions

\( I \) Will be minimum when \( \sin \alpha = 0 \)

\[
\alpha = \pm m\pi \quad m = 1, 2, 3, 4, 5 \ldots \ldots
\]
\[ \alpha = \frac{\pi e \sin \theta}{\lambda} = \pm m\pi \]
\[ \pi e \sin \theta = \pm m\lambda \]

So we obtain the minimum intensity positions on either side of the principal maxima for all \( \alpha = \pm m\pi \) values.

**Secondary maximum**

In between these minima secondary maxima positions are located. This can be obtained by differentiating the expression of \( I \) w.r.t \( \alpha \) and equation to zero

\[ \frac{dI}{d\alpha} = \frac{d}{d\alpha} \left( A^2 \left[ \frac{\sin \alpha}{\alpha} \right]^2 \right) = 0 \]
\[ A^2 \frac{2\sin \alpha}{\alpha} \frac{d}{d\alpha} \left( \frac{\sin \alpha}{\alpha} \right) = 0 \]
\[ A^2 \frac{2\sin \alpha}{\alpha} \cdot \left[ \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} \right] = 0 \]

In the above expression \( \alpha \) can never equal to zero, so

Either \( \sin \alpha = 0 \) or \( \alpha \cos \alpha - \sin \alpha = 0 \)

\( \sin \alpha = 0 \) gives the positions of minima.

The condition for getting the secondary maxima is

\[ \alpha \cos \alpha - \sin \alpha = 0 \]
\[ \alpha \cos \alpha = \sin \alpha \]
\[ \alpha = \tan \alpha \]

The values of \( \alpha \) satisfying the above equation are obtained graphically by plotting the curves \( Y = \alpha \) and \( Y = \tan \alpha \) on the same graph. The plots of \( Y = \alpha \) and \( Y = \tan \alpha \) is shown in figure.
In the graph the two curves intersecting curves gives the values of satisfying of $\alpha$ satisfying the above equation. From the graph intersecting points are $\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$ ...........

Substituting the values of $\alpha$ in the Intensity equation, we get the intensities at various maxima.

$\alpha = 0, I_0 = A^2$ (principal maximum)

$\alpha = \frac{3\pi}{2}, I_1 = \frac{A^2}{22}$ 1st secondary maximum

$\alpha = \frac{5\pi}{2}, I_2 = \frac{A^2}{62}$ 2nd secondary maximum

From the above concepts I vs. $\alpha$ is shown in figure.
4. Fraunhofer double slit diffraction

Let us consider two slits $S_1$ and $S_2$ having equal width $e$ and separated by a distance $d$. The distance between the two slits is $(e + d)$. Let a plane wave front of monochromatic light of wavelength $\lambda$ be incident on the two slits. The diffracted light from these slits is focused on the screen by using a lens. The diffraction of a two slits is a combination of diffraction and interference. When the plane wave front is incident on the two slits, the secondary wavelets from these slits travel in all directions. The wavelets travelling perpendicular to the slit is focused at point $P_0$. The wavelets travelling at an angle $\theta$ with the incident light are focused at point $P_1$.

From Fraunhofer single slit experiment, the resultant amplitude is $R = A \sin \alpha / \alpha$.

So the amplitude of each secondary wavelet travelling with an angle $\theta$ can be taken as $A \sin \alpha / \alpha$.

These two wavelets interference and meet at point $P_1$ on the screen.

To find out the path difference between the two wavelets, let us draw a normal $s_1 k$ to the wavelet $S_2$.

Path difference $= S_2 k$

From $\Delta S_1 S_2 k$ $\sin \theta = \frac{s_2 k}{s_1 S_2} = \frac{s_2 k}{(e + d)}$

Path difference $= S_2 k = (e + d) \sin \theta$

Phase difference $= \frac{2\pi}{\lambda}$ (path difference)

Phase difference $= \frac{2\pi}{\lambda} (e + d) \sin \theta = \delta$

By using vector addition method, we can calculate the resultant amplitude at point $P_1$ by taking the resultant amplitudes of the two slits $S_1$ and $S_2$ as sides of the triangle. The third side gives resultant amplitude.

From the figure

$AC^2 = AB^2 + BC^2 + 2(AB)(BC) \cos \delta$

$R^2 = \left(A \sin \frac{\alpha}{\lambda}\right)^2 + \left(A \sin \frac{\alpha}{\lambda}\right)^2 + 2\left(A \sin \frac{\alpha}{\lambda}\right)\left(A \sin \frac{\alpha}{\lambda}\right) \cos \delta$

$R^2 = 2\left(A \sin \frac{\alpha}{\lambda}\right)^2 + 2\left(A \sin \frac{\alpha}{\lambda}\right)^2 \cos \delta$

$= 2\left(A \sin \frac{\alpha}{\lambda}\right)^2 + 2\left(A \sin \frac{\alpha}{\lambda}\right)^2 \cos \delta$
\[ R^2 = 2 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \left[ 1 + \cos \delta \right] \]
\[ R^2 = 2 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \left[ 1 + 2 \cos^2 \frac{\delta}{2} - 1 \right] \]
\[ R^2 = 2 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \left[ 2 \cos^2 \frac{\delta}{2} \right] \]
\[ R^2 = 4 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \left[ \frac{2\pi(e+d)\sin \theta}{\lambda} \right] \]
\[ R^2 = 4 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \beta \]

Since \( \beta = \frac{\pi(e+d)\sin \theta}{\lambda} \)

The resultant intensity \( I = R^2 = 4 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \beta \)

From the above equation, it is clear that the resultant intensity is a product of two factors i.e.,

1. \( \left( A \frac{\sin \alpha}{\alpha} \right)^2 \)
   Represents the diffraction pattern due to a single slit.

2. \( \cos^2 \beta \)
   Represents the interference pattern due to wavelets from double slit.

**Diffraction effect**

In diffraction pattern the variation of \( I \) w.r.t \( \alpha \) as shown in figure.

The diffraction pattern consists of central principal maximum for \( \alpha = 0 \) and \( \theta = 0 \) value.

The secondary maximum of decreasing intensity is present on either side of the central maxima for \( \alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2} \) values.

Between the secondary maxima the minima values are present for \( \alpha = \pm \pi, \pm 2\pi, \pm 3\pi \) values.
Interference effect

In interference pattern the variation of $\cos^2 \beta$ w.r.t $\beta$ as shown in figure. $\cos^2 \beta$ represents the interference pattern.

Interference maximum will occur for $\cos^2 \beta = 1$

$$\beta = \pm m\pi \quad \text{where} \quad m = 0, 1, 2, 3, 4 \ldots \ldots$$

$$\beta = \pm \pi, \pm 2\pi, \pm 3\pi, \pm 4\pi \ldots \ldots$$

$$\frac{\pi (e + d) \sin \theta}{\lambda} = \pm m\pi$$

$$\pi (e + d) \sin \theta = m\lambda$$

Interference minima will occur for $\cos^2 \beta = 0$

$$\beta = \pm (2m + 1) \frac{\pi}{2}$$

$$\frac{\pi (e + d) \sin \theta}{\lambda} = \pm (2m + 1) \frac{\pi}{2}$$

$$\pi (e + d) \sin \theta = \pm (2m + 1) \frac{\lambda}{2}$$

The resultant intensity variation due both interference and diffraction patterns is shown in figure. Due to interference the resultant minimum in not exactly equal to zero.

$$I = 4 \left( A \frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \beta$$
5. Diffraction grating

Definition:- A set of large number of parallel slits of same width and separated by opaque spaces is known as diffraction grating.

Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular separation. Now the gratings are constructed by ruling the equidistance parallel lines on a transparent material such as glass with fine diamond point. The ruled lines are opaque to light while the space between the two lines is transparent to light and act as a slit.

Let 'e' be the width of line and 'd' be the width of the slit. Then \((e + d)\) is known as grating element. If \(N\) is the number of lines per inch on the grating then

\[
N(e + d) = 1 \text{ inch} = 2.54 \text{ cm}
\]

\[
(e + d) = \frac{2.54 \text{ cm}}{N}
\]

Commercial gratings are produced by taking the cost of actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on a ruled surface and allowed to dry to form a thin film, detachable from the surface. This film of grating is kept between the two glass plates.